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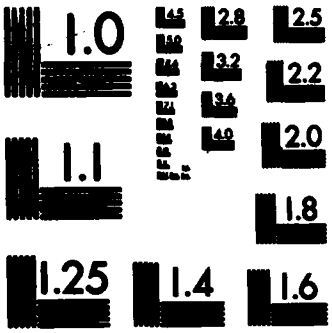

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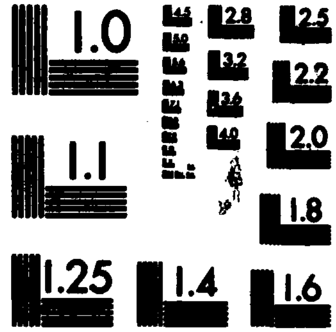

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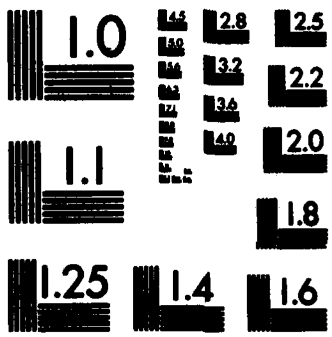

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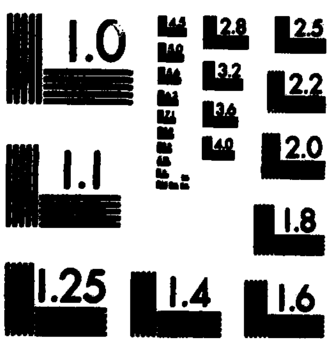

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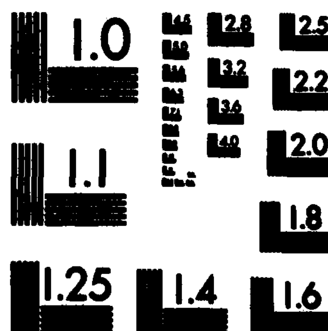

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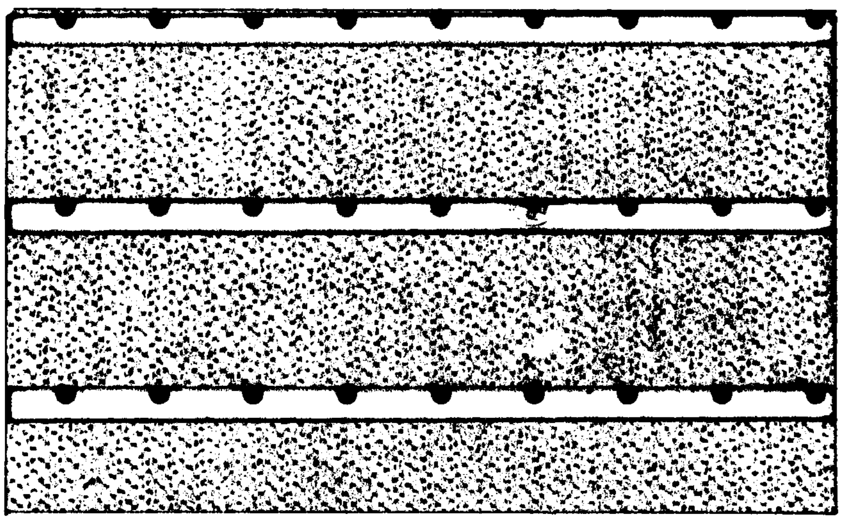

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ERROR MODELS FOR STABLE
HYBRID ADAPTIVE CONTROL

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Error Models for Stable Hybrid Adaptive Control

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1. Introduction: In the past few years several continuous [1,2] and discrete [3,4] adaptive algorithms have been developed for the stable control of linear time-invariant systems with unknown parameters. Recently Gawthrop [5] introduced the concept of hybrid self tuning controllers which are partly realized in continuous time and partly in discrete time. In [6] Elliott presented a method for the discrete indirect adaptive control of a continuous time system and more recently an attempt was also made by Cristi and Monopoli [7] for the direct control of model reference adaptive systems. In both [6] and [7] the process to be controlled is continuous while the adaptive algorithm for updating the control parameters is discrete. The central question in both cases is the stability of the overall system. As in the case of purely continuous and purely discrete systems this can be best analyzed by considering the stability of an error model. For the hybrid adaptive control problem this may be described by an error equation (algebraic or differential) in which the parameters are adjusted in a discrete fashion.

The main objective of this brief report is to analyze the asymptotic behavior of three error models which arise in the identification and control of hybrid adaptive systems. Using methods developed in [1] and [3] this analysis can be applied directly to establish the global stability of hybrid model reference adaptive systems. These results as well as the design considerations which arise in such problems will be contained in a forthcoming paper [8].

2. <u>Error Models</u>: The dynamical systems discussed in the following section are continuous time systems in which $t \in R^+$, the set of positive real numbers. Let $u: R^+ \to R^m$ and $e_1: R^+ \to R$ be piecewise continuous functions, referred to as the input and output error functions respectively. Let $\{t_n\}$ be a monotonically increasing unbounded sequence in R^+ with $0 < T_{min} \leq |t_n - t_{n-1}| \leq T_{max} < \infty$ where $n \in N$, the set of positive integers. Finally, let $\phi: R^+ \to R^m$ be a piecewise constant function referred to as the parameter error and assume values

$$\phi(t) = \phi_k \qquad t \in [t_{k-1}, t_k)$$

where ϕ_k is a constant vector. The error models described in this report relate the error e_1, the input u and parameter error ϕ in terms of algebraic or differential equations.

a) <u>Error Model 1</u>: The first hybrid error model is described by the equation

$$\phi_k^T u(t) = e_1(t) \qquad t \in [t_{k-1}, t_k) \quad \forall\ k \in N \tag{1}$$

The objective is to determine an adaptive law for choosing the sequence $\{\phi_k\}$ so that $\lim_{t\to\infty} e_1(t) = 0$.

Consider the Lyapunov function candidate

$$V(k) = 1/2\ \phi_k^T \phi_k . \tag{2}$$

Then

$$\begin{aligned} \Delta V(k) &\triangleq V(k+1) - V(k) \\ &= [\phi_k^T + \frac{\Delta\phi_k^T}{2}]\Delta\phi_k \end{aligned} \tag{3}$$

where $\Delta\phi_k \triangleq \phi_{k+1} - \phi_k$. Choosing the adaptive law

$$\Delta\phi_k = -\,1/T_k \int_{t_{k-1}}^{t_k} \frac{e_1(\tau)u(\tau)}{1 + u^T(\tau)u(\tau)}\, d\tau \tag{4}$$

where $T_k \triangleq (t_k - t_{k-1})$, yields

$$\Delta V(k) = -\,1/2\ \phi_k^T[2I - R_{k-1,k}]R_{k-1,k}\phi_k \leq 0 \tag{5}$$

where $R_{k-1,k}$ is the positive semi-definite symmetric matrix

$$R_{k-1,k} = 1/T_k \int_{t_{k-1}}^{t_k} \frac{u(\tau)u^T(\tau)}{1 + u^T(\tau)u(\tau)}\, d\tau\ . \tag{6}$$

Hence $V(k)$ is a Lyapunov function and assures the boundedness of $\| \phi_k \|$ if $\| \phi_0 \|$ is bounded.

<u>Case (i)</u>: If u is uniformly bounded in $[0,\infty)$ it follows from (1) that e_1 is also uniformly bounded. Since $[2I - R_{k-1,k}] > \beta R_{k-1,k}$ for some constant $\beta > 0$ it follows that

$$\Delta V(k) < -\,\beta\phi_k^T\, R_{k-1,k}\phi_k \leq 0 \tag{7}$$

and hence $\Delta V(k) \to 0$ as $k \to \infty$. Since

$$\Delta V(k) < -\frac{\beta}{T_k}\int_{t_{k-1}}^{t_k} \frac{e_1^2(\tau)}{1 + u^T(\tau)u(\tau)}\, d\tau \leq 0\ ,$$ this implies that $e_1 \in L^2$.

If $|\dot{u}|$ is also bounded it follows $\lim_{t\to\infty} e_1(t) = 0$. Hence for a uniformly bounded input u with a bounded derivative e_1 tends to zero and $\Delta\phi_k \to 0$ as $k \to \infty$.

<u>Case ii</u>: If in addition to being uniformly bounded, $u(\cdot)$ is "sufficiently rich" over an interval T_{min} so that $R_{k-1,k}$ is positive definite for all $k \in N$, $\Delta V(k) < 0$ and hence the parameter error vector ϕ tends to zero as $k \to \infty$.

<u>Case iii</u>: A more interesting case arises in the control problem where u may grow in an exponential fashion. Since condition (7) is independent of the assumption of boundedness on u,

$$\int_{t_{k-1}}^{t_k} \frac{e_1^2(\tau)}{1 + u^T(\tau)u(\tau)} d\tau \to 0 \quad \text{as } k \to \infty$$

and hence $e_1(t) = o\,\|u(t)\|$ or e_1 grows slowly [1] compared to the norm of the input vector u. [In the control problem this leads to a contradiction, assuring the boundedness of the plant to be controlled].

b) Error Model 2:

The second error model is described by the error differential equation

$$\dot{e} = Ae + b\phi^T u \tag{8}$$

where $e(t)$, $b \in R^n$, $A \in R^{n\times n}$ and is stable, $\phi(t)$, $u(t) \in R^m$ and $\phi(t) = \phi_k$ $t \in [t_{k-1}, t_k)$, $k \in N$. In this case the parameter error ϕ is to be adjusted using the input $u(t)$ and the state error vector $e(t)$. Since A is a stable matrix a symmetric positive definite matrix $P = P^T > 0$ exists such that $A^TP + PA = -Q < 0$. The time derivative of the quadratic form $e^T(t)Pe(t)$ may be expressed as

$$\frac{d}{dt} e^T(t)Pe(t) = -e^T(t)Qe(t) + 2e^T(t)Pb\phi_k^T u(t) \qquad t \in [t_{k-1}, t_k) \tag{9}$$

Integrating (9) over the interval $[t_{k-1}, t_k]$ we obtain

$$e^T(t)Pe(t)\Big|_{t_{k-1}}^{t_k} + \int_{t_{k-1}}^{t_k} e^T(\tau)Qe(\tau)d\tau = 2\phi_k^T \int_{t_{k-1}}^{t_k} e^T(\tau)Pbu(\tau)d\tau \tag{10}$$

Defining

$$e^T(t)Pe(t)\Big|_{t_{k-1}}^{t_k} + \int_{t_{k-1}}^{t_k} e^T(\tau)Qe(\tau)d\tau \triangleq \varepsilon_1(k) \tag{11a}$$

and
$$2\int_{t_{k-1}}^{t_k} e^T(\tau)Pbu(\tau)d\tau = \bar{u}(k) \tag{11b}$$

equation (10) may be expressed as

$$\phi_k^T\bar{u}(k) = \varepsilon_1(k) \tag{12}$$

which is the discrete error model of type 1 well described in the literature. By inspection the adaptive law for updating ϕ can be written as:

$$\phi_{k+1} - \phi_k = \Delta\phi_k = \frac{-\epsilon_1(k)\bar{u}(k)}{1 + \bar{u}^T(k)\bar{u}(k)} \tag{13}$$

and directly yields the boundedness of ϕ_k for all $k \in N$.

If u in equation (8) is uniformly bounded, it follows from (8) and (13) that e(t) is also uniformly bounded. From (11a), (11b) it follows that $\epsilon_1(k)$ and $\bar{u}(k)$ are bounded for all $k \in N$. Hence, by well known results in discrete adaptive systems [3] we have $\epsilon_1(k)$ and $\Delta\phi_k \to 0$ as $k \to \infty$. $\lim_{k\to\infty} \epsilon_1(k) = 0$ implies by (11a) that $\lim_{t\to\infty} e(t) = 0$. If u is sufficiently rich on any interval of length T_{min}, e(t) can tend to zero only if $\phi_k \to 0$ in equation (8). Hence the parameter errors converge to zero in such a case.

For the important case when u(t) grows at most exponentially, by equation (8) e(t) can also grow at most exponentially at the same rate. From equation (13) we have

$$\frac{\epsilon_1(k)}{1 + \|\bar{u}(k)\|} \to 0 \text{ or } \epsilon_1(k) = o\|\bar{u}(k)\| . \tag{14}$$

Since Q is positive definite and e is a continuous function, from (11a) and (11b) we can conclude that $e(t) = o\|u(t)\|$ or the output error grows slowly compared to the norm of the input vector. [As in the previous case when this model is used in the control problem the boundedness of the plant output can be proved by contradiction.]

c) <u>Error Model 3</u>: In contrast to the second error model, the third error model contains a positive real transfer function but with only the output (rather than the entire state vector) accessible. The general hybrid third error model is described by

$$\dot{e}(t) = Ae(t) + b\phi_k^T u(t) \qquad t \in [t_{k-1}, t_k)$$
$$e_1(t) = c^T e(t) \qquad k \in N \tag{15}$$

where $W(s) = c^T(sI-A)^{-1}b$ is a strictly positive real (SPR) transfer function. The parameter error vector ϕ is to be updated at t_k using only the values of the input $u(t)$ and the output $e_1(t)$ over the interval $[t_{k-1}, t_k)$. In the following analysis we assume that $W(s) = 1/(s+\alpha)$ i.e. a first order transfer function. Since any (SPR) transfer function can be reduced to a first order transfer function by post multiplication by a stably invertible transfer function $T(s)$, the method outlined can be extended to any general error model of type 3.

If the hybrid error model 3 is described by

$$\dot{e}_1(t) = -\alpha e_1(t) + \phi_k^T u(t) \qquad t \in [t_{k-1}, t_k)$$
$$k \in N \tag{16}$$

the results derived for error model 2 can be extended directly to this case also.

Equation (11a) and (11b) can be specialized to

$$1/2\, e_1^2(t)\Big|_{t_{k-1}}^{t_k} + \alpha \int_{t_{k-1}}^{t_k} e_1^2(\tau)d\tau \triangleq \varepsilon_1(k)$$
$$\int_{t_{k-1}}^{t_k} e_1(\tau)u(\tau)d\tau \triangleq \bar{u}(k)$$

resulting in the equation

$$\phi_k^T\, \bar{u}(k) = \varepsilon_1(k).$$

The adaptive law

$$\Delta\phi_k = \frac{-\varepsilon_1(k)\bar{u}(k)}{1 + \bar{u}^T(k)\bar{u}(k)}$$

results in (using the same arguments as before)

(i) $\epsilon_1(k)$, and hence $e_1(t) \to 0$ as $k \to \infty$, $t \to \infty$ and $\Delta\phi_k \to 0$ as $k \to \infty$ if u is uniformly bounded

(ii) if u is sufficiently rich $\lim_{k\to\infty} \phi_k = 0$

(iii) if $\|u(t)\|$ grows at most exponentially, $e_1(t)$ grows at a slower rate than $\|u(t)\|$ i.e. $e_1(t) = o\,\|u(t)\|$.

<u>Comments:</u>

1. The adaptive laws for error models 2 and 3 were derived by sampling the continuous time Lyapunov functions (rather than discretizing the differential equations [7]).

2. In all three cases the entire observed data is used in the generation of the adaptive laws.

3. The principal question in the hybrid control problem is whether the plant output $y_p(t)$ will remain bounded. This can be directly established using the results of the preceding analysis. This will be presented in a forthcoming paper [8] along with a discussion regarding speed of convergence.

4. In the hybrid third error model our analysis applies only when $W(s)$ is a first order transfer function. The more general case remains an open problem. As indicated in section 2c, since every (SPR) transfer function can be reduced to a transfer function of first order, the results can be extended to any error model of type 3.

5. Adaptive gains have not been included in the adaptive laws derived in section 2 to avoid obscuring the principal results. The effective use of such gains to improve transient response will be discussed in [8].

3. <u>Conclusion:</u> It is shown in this brief note that by suitably reducing hybrid error models into equivalent discrete error models of type 1 all known adaptive algorithms can be extended directly to the hybrid adaptive control problem.

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